## INVESTIGATION OF THE INFLUENCE OF PRESSURE FLUCTUATIONS OF THE HEAT CARRYING AGENT ON THE MEAN HEAT-TRANSFER COEFFICIENT IN A TUBE

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Experimental heat-transfer data averaged over the tube length are examined for pulsating air flows through the tube. It is shown that maximum changes in heat transfer occur at resonant frequencies. Dimensionless relations which generalize the mean heat-transfer data are obtained.

Experimental data on local heat-transfer coefficients obtained for various pressure fluctuation frequencies near the second resonance harmonic  $f_s \approx 180 \text{ Hz}$  (135 Hz  $\leq f \leq 225 \text{ Hz}$ ) were obtained in [1, 2].

In the present paper, which is a continuation of [1-3], we examine the influence of resonance and nonresonance pressure fluctuations of the heat-transfer agent on the heat-transfer coefficient averaged over the tube length.

The experimental facility employed in [1, 2] was also used in our experiments. An electrically heated stainless steel tube with an inner diameter of 9.7 mm, a wall thickness of 0.65 mm, and a length of 1855 mm (usable length).

The air, which served as the heat-transfer agent, was supplied to the usable length by a pulser which produced the pressure fluctuations. A rotor with radial slits served as the pulser. The amplitude of the pressure fluctuations was measured with an ID-2I induction pressure gauge at the inlet and outlet of the usable length.

The wall temperature was measured with 18 Chromel-Alumel thermocouples, welded to the outer surface of the test tube, which measured 0.1 mm in diameter and whose hot junctions were spaced 100 mm apart.

The heat flow was determined from the current strength and from the heat leakage to the thermal insulation about the usable length. The time-averaged air flow rate was measured with a conventional orifice.

The heat-transfer coefficient averaged over the tube length was determined from the formula  $\overline{\alpha} = \overline{q}_{w}$ .  $/(\overline{T}_w - \overline{T}_f)$ .

The physical properties of the gas, which are contained in the dimensionless numbers (Nu, Re, Pr), were determined from the mean transverse and longitudinal gas temperature  $\overline{T}_{f}$ . A detailed description of the experimental facility and procedure is to be found in [1].

The principal parameters were measured within the following ranges: Reynolds number  $\text{Re} = 10^4$  to  $10^5$ , pressure P = 3 to 20 bar, the relative amplitude of the pressure fluctuations at the inlet of the test tube  $(\Delta P/P)_0 = 0$  to 0.25, and the temperature factor  $T_W/T_f = 1.2$  to 1.5. The frequency of the pressure fluctuations was measured within the range f = 40 to 500 Hz. The resonant frequencies  $f_S = 90$ , 180, 270, 360, and 450 Hz corresponded to the first, second, third, fourth, and fifth resonance harmonics of a channel acoustically sealed at both ends.

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Fig. 1. Distribution of local relative heat transfer  $K_s = Nu_s / Nu_0$  over the tube length (x) for a pressure fluctuation frequency  $f_s = 450$  Hz that corresponds to the fifth resonance harmonic, for various values of the relative pressure-fluctuation amplitude at the inlet: 1)  $(\Delta P/P)_{0S} = 0.165$ ; 2) 0.086; 3) 0.035; 4) 0.0275.

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where

$$\varphi = \frac{\eta_L}{\eta_{\lambda/2}} \ln \frac{\left(\frac{\Delta E_{\kappa}}{E_0}\right)_{\max,i}}{\left(\frac{\Delta E_{\kappa}}{E_0}\right)_{\max,i+1}}$$

is the damping factor of the fluctuation energy over the tube length.

The heat transfer damping factor averaged over the tube length

$$\widetilde{\psi} = \frac{1}{n} \sum_{i=1}^{n} \psi_i, \text{where } \psi_i = \frac{\eta_{L_i}}{\eta_{\lambda/2}} \ln \frac{K_{\max,i}}{K_{\max,i+1}}$$

is almost independent of the number of the resonance harmonic and of the Reynolds number, while its numerical value coincides within an error of  $\pm 7\%$  with the mean damping factor of the fluctuation energy  $\psi \simeq \overline{\varphi}$ , and is generalized by the relation

$$\overline{\psi} = \overline{\varphi} = 1.06 \left[ 1 - \exp\left\{ -10 \left( \frac{\Delta P}{P} \right)_{0s} \right\} \right].$$
(3)

Since, for resonance fluctuations, the distribution of the local relative heat transfer  $K_s = Nu_s / Nu_0$  is similar to that of the kinetic energy of the fluctuations in the length of the standing wave [1], the relative heat transfer averaged over the tube length must also depend on the kinetic energy of the fluctuations averaged over the length of the standing wave.

The mean kinetic energy of the fluctuations is determined from Eq. (2)

$$\left(\frac{\overline{\Delta E}_{\mathbf{R}}}{E_{0}}\right)_{n} = \frac{\left(\frac{\Delta E_{\mathbf{R}}}{E_{0}}\right)_{\max \mathbf{1}}}{\exp\left(-\frac{\overline{\varphi}}{2n}\right)} \int_{0}^{1} \exp\left(-\overline{\varphi} \frac{\eta_{x}}{\eta_{L}}\right) \sin^{2}\left(\pi n \frac{\eta_{x}}{\eta_{L}}\right) d\left(\frac{\eta_{x}}{\eta_{L}}\right) = \left(\frac{\Delta E_{\mathbf{R}}}{E_{0}}\right)_{\max \mathbf{1}} \frac{1}{2\overline{\varphi}} \frac{1 - \exp\left(-\overline{\varphi}\right)}{\left[1 + \left(\frac{\overline{\varphi}}{2\pi n}\right)^{2}\right] \exp\left(-\frac{\overline{\varphi}}{2n}\right)},$$
(4)

Stationary heat transfer was determined experimentally prior to each series of nonstationary tests. The stationary heat-transfer data are in satisfactory agreement with standard recommendations, and are generalized with an error of  $\pm 6\%$ by the dimensionless equation

$$\overline{\mathrm{N}}\mathrm{u}_{0} = 0.021 \ \overline{\mathrm{Re}}^{0.8} \ \overline{\mathrm{Pr}}^{0.4} \left(\frac{\overline{T}_{w}}{\overline{T}_{f}}\right)^{-0.5}$$
(1)

In [1-3], it was shown that in the case of resonance air-pressure fluctuations in tubes near the velocity antinodes (pressure nodes) of a standing wave, the heat transfer coefficient has its maximum value, while its value is minimum near the velocity nodes (pressure antinodes). The number of heat-transfer maxima and minima is defined by the number of the resonance harmonic. Thus, for a fluctuation frequency corresponding to the fifth resonance harmonic (Fig. 1), five heat-transfer maxima are distributed over the tube length. The local heat-transfer distribution over the tube length is qualitatively similar to that of the kinetic energy of the fluctuations in the length of the standing wave [1]

$$\frac{\Delta E_{\rm R}}{E_0} = \frac{1}{2k} \left(\frac{\Delta P}{P}\right)_{0s}^2 \exp\left(-\phi \frac{\eta_x}{\eta_L}\right) \sin^2\left(\pi n \frac{\eta_x}{\eta_L}\right) \quad (2)$$



Fig. 2. Mean relative heat transfer  $\overline{K}_S$  vs the relative amplitude of the pressure fluctuations at the tube inlet  $(\Delta P/P)_{0S}$  at various resonant frequencies: a, 1)  $f_S = 90 \text{ Hz}$ , n = 1; b, 2)  $f_S = 180 \text{ Hz}$ , n = 2; c, 3)  $f_S$ = 270 Hz, n = 3; d, 4)  $f_S = 360 \text{ Hz}$ , n = 4; e, 5)  $f_S$ = 450 Hz, n = 5. Solid curves correspond to Eq. (9).



Fig. 3. Variation of the mean relative heat transfer and the relative amplitude of the pressure fluctuation as a function of a deviation of the fluctuation frequency from the resonance frequency ( $45 \text{ Hz} \le f \le 315 \text{ Hz}$ ): 1) relative heat transfer K; 2) relative amplitude of the pressure fluctuation A<sub>0</sub>.

where

$$\left(\frac{\Delta E_{\rm K}}{E_0}\right)_{\rm max^1} = \frac{1}{2k} \left(\frac{\Delta P}{P}\right)_{\rm os}^2 \exp\left[-\bar{\varphi} \left(\frac{\eta_x}{\eta_L}\right)_{\rm max^1}\right]$$
$$= \frac{1}{2k} \left(\frac{\Delta P}{P}\right)_{\rm os}^2 \exp\left(-\frac{\bar{\varphi}}{2n}\right)$$

is the kinetic energy of the fluctuations at the i-th velocity antinode.

The relative heat transfer at the first antinode of the standing wave velocity depends on the relative amplitude of the pressure fluctuations  $(\Delta P/P)_{0S}$ , and is almost independent of the number of the resonance harmonic [3]; consequently, it may be assumed that  $(\Delta E_K/E_0)_{max1}$  is also independent of the number of the resonance harmonic.

Bearing in mind that the mean damping factor  $\overline{\varphi}$  and  $(\Delta E_{\rm K}/E_0)_{\rm max_1}$  are independent of the number of the resonance harmonic, from expression (4) we obtain

$$\frac{\left(\frac{\overline{\Delta E}_{R}}{\overline{E}_{0}}\right)_{n}}{\left(\frac{\overline{\Delta E}_{R}}{\overline{E}_{0}}\right)_{n=1}} = \frac{1 + \left(\frac{\overline{\varphi}}{2\pi}\right)^{2}}{1 + \left(\frac{\overline{\varphi}}{2\pi n}\right)^{2}} \exp\left[-\frac{\overline{\varphi}}{2} \frac{n-1}{n}\right].$$
 (5)

In our test, the mean damping factor  $\overline{\varphi}$  varied from  $\leq 1$  to 1.06. Within this range, the quantities  $(\overline{\varphi}/2\pi)^2$  and  $(\varphi/2\pi n)^2$  are much smaller than unity, and expression (5) can be expressed with satisfactory accuracy (~2%) in the form

$$\frac{\left(\frac{\overline{\Delta E_{\rm R}}}{E_0}\right)_n}{\left(\frac{\overline{\Delta E_{\rm R}}}{E_0}\right)_{n=1}} \simeq \exp\left[-\frac{\overline{\varphi}}{2} \frac{n-1}{n}\right]. \tag{6}$$

By assuming that at fixed values of the Reynolds number, Prandtl number, and the temperature factor  $T_W/T_f$ , the mean relative heat transfer  $\overline{K}_S$  is proportional to the mean kinetic energy of the fluctuations  $\overline{K}_S - 1 \sim \langle \overline{\Delta E}_K / E_0 \rangle$ , we obtain a relation between the mean heat transfer and the number of the resonance harmonic

$$\frac{\overline{K}_{sn}-1}{\overline{K}_{s1}-1} = \exp\left[-\frac{\overline{\varphi}}{2} \cdot \frac{n-1}{n}\right],\tag{7}$$

where  $\overline{K_{S1}}$  is the mean relative heat transfer at a frequency corresponding to the first resonance harmonic.

From condition (7), it follows that for fixed Re, Pr, and  $T_W/T_f$ , owing to the damping of the kinetic energy of the fluctuations, the relative heat transfer decreases with increasing number of the resonance harmonic, and at large values of  $n \rightarrow \infty$ , tends to the limit

$$\overline{K}_{s,n=\infty} - 1 = (\overline{K}_{s1} - 1) \exp\left[-\frac{\varphi}{2}\right]$$
(8)

Thus, at large values of  $n \rightarrow \infty$ , the mean relative heat transfer is almost independent of the resonance harmonic.

Equation (7) for the mean heat transfer is in satisfactory agreement with experimental data. Figure 2 shows processed mean heat transfer data for resonance fluctuations of air pressure in a tube acoustically sealed at both ends. From Fig. 2 it can be seen that the mean relative heat transfer increases when the relative amplitude of the pressure fluctuations at the inlet  $(\Delta P/P)_{0S}$  increases. For large values of  $(\Delta P/P)_{0S} \simeq 0.25$ , the mean heat transfer at n = 1 increases by 40% of its stationary value. For a constant value of  $(\Delta P/P)_{0S}$ , the mean heat transfer decreases when the number of the resonance harmonic increases (for  $(\Delta P/P)_{0S} = 0.25$  at n = 4, and we have  $\overline{K}_{S} = 1.28$ ). This may probably be attributed to the dissipation of the fluctuation energy along the tube length. In the ideal case, when the energy of the fluctuations is not damped ( $\varphi = 0$ ), the mean heat transfer, according to (7), should not depend on the number of the resonance harmonic.

For all five resonant frequencies  $f_s = 90, 180, 270, 360, 450$  Hz that correspond to the first, second, third, fourth, and fifth resonance harmonics, the mean relative heat transfer can be satisfactorily (with an error of  $\pm 7\%$ ) generalized by Eq. (7), i.e.,

$$\overline{K}_{sn} = 1 + 1.77 \left(\Delta P/P\right)_{0s} \exp\left(-\frac{\varphi}{2} \frac{n-1}{n}\right), \qquad (9)$$

where  $\overline{\varphi}$  depends on  $(\Delta P/P)_{0S}$ , and is determined from Eq. (3).

When the fluctuation frequency differs from the resonant frequency, the heat transfer is smaller than in the resonance mode.

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Figure 3 experimental mean heat-transfer data can be expressed in the form:

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$$\frac{K-1}{\bar{K}_{s}-1} = \Phi_{1} \left(n \frac{\Delta f}{f_{s}}\right),$$

$$A_{0} = \frac{\left(\frac{\Delta P}{P}\right)_{0} - \left(\frac{\Delta P}{P}\right)_{\text{ot}}}{\left(\frac{\Delta P}{P}\right)_{0} - \left(\frac{\Delta P}{P}\right)_{\text{ot}}} = \Phi_{2} \left(n \frac{\Delta f}{f_{s}}\right),$$

$$\Delta f = f_{s} - f = \pm \frac{1}{2n} f_{s} = \pm 45 \text{ Hz}^{*}).$$

It can be seen from Fig. 3 that at a maximum deviation of the pressure fluctuation frequency from the resonant frequency,  $\Delta f = \pm 45$  Hz, the amplitude of the pressure fluctuations  $(\Delta P/P)_0$  and, consequently, the energy of the fluctuations reach a minimum value  $(\Delta P/P)_0 = (\Delta P/P)_{ot}$ , while the mean relative heat transfer  $\bar{K}$  does not differ practically from the corresponding stationary value  $\bar{K} \simeq 1$ . From this figure, it can be further seen that a change in the relative heat transfer  $(\bar{K}-1)/(\bar{K}_S-1)$  as a function of the magnitude of the deviation from the resonance  $n \cdot \Delta f/f_S$  is qualitatively similar to the change in the dimensionless amplitude of the pressure fluctuation at the tube inlet  $A_0$  (Fig. 3). The experimental data on mean heat transfer for all the nonresonant frequencies studied in this paper can be satisfactorily (with an error of  $\pm 7\%$ ) generalized by the dimensionless equation

$$\bar{K} = 1 + (\bar{K}_s - 1) A_0.$$
 (10)

<sup>\*</sup>The maximum deviation of the pressure fluctuation frequency from the resonant frequency for a tube acoustically sealed at both ends corresponds to the resonant frequency of a tube acoustically open at one end.

Re. Pr are the Reynolds and Prandtl numbers averaged over the tube length; Nu is the stationary value of the Nusselt number averaged over the tube length;  $\overline{T}_w, \overline{T}_f, \overline{q}_w$ are the wall and heat-transfer-agent temperatures averaged over the tube length, and the heat flow, respectively;  $\mathbf{P}, \Delta \mathbf{P}$ are the pressure, and amplitude of the pressure fluctuations of the heattransfer agent, respectively;  $(\Delta P/P)_{0S}, (\Delta P/P)_0$ are the relative amplitudes of the pressure fluctuation at the tube inlet at resonant and nonresonant frequencies, respectively;  $(\Delta P / P)_{ot}$ is the relative amplitude of the pressure fluctuation at the tube inlet at the maximum deviation of the pressure fluctuation frequency from the resonant frequency; α is the heat-transfer coefficient averaged over the tube length; Nu<sub>S</sub>, Nu are the Nusselt numbers averaged over the tube length at resonance and nonresonance pressure fluctuations, respectively;  $\overline{\mathbf{K}} = \mathrm{Nu}/\mathrm{Nu}_0, \, \overline{\mathbf{K}}_{\mathbf{S}} = \mathrm{Nu}_{\mathbf{S}}/\mathrm{Nu}_0$ are the mean values of the relative heat-transfer coefficients at resonant and nonresonant frequencies (here, the Nusselt numbers refer to the same Re, Pr, and  $T_w/T_f$ ;  $\Delta E_{K} = \rho \Delta U^{2}/2, E_{0} = P_{0}$ are the amplitude of the kinetic energy of the fluctuations and the potential energy of the pressure in a steady flow of the heat-transfer agent, respectively:

are the ratio of specific heats, density, speed of sound, and amplitude of the heat-transfer-agent velocity fluctuations, respectively;

are the instantaneous coordinate and total tube length, respectively;

are the resonant and nonresonant frequency, respectively;

is the deviation of the fluctuation frequency from the resonant frequency; are the length of the standing wave, number of the resonance harmonic, and order of the velocity antinode, respectively.

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## NOTATION

$$\eta_{\mathbf{X}} = \int_{0}^{x} d\mathbf{x}/a; \ \eta_{\mathbf{L}} = \int_{0}^{L} d\mathbf{x}/a;$$
  
k, L  
$$\mathbf{f}_{\mathbf{S}}, \mathbf{f}$$
  
$$\Delta \mathbf{f} = \mathbf{f}_{\mathbf{S}} - \mathbf{f}$$

k,  $\rho$ , a,  $\Delta U$ 

 $\lambda, n, i$ 

$$\eta_{\mathbf{X}} = \int_{0}^{\mathbf{X}} d\mathbf{x} / a; \ \eta_{\mathbf{L}} = \int_{0}^{L} d\mathbf{x} / a;$$
  
x, L  
f<sub>s</sub>, f